Quantum-Powered Logistics: Towards an Efficient and Sustainable Supply Chain

Airbus - BMW Group
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1 Introduction

The drive towards sustainable production makes the need to minimize emissions in the supply chain a priority concern. In particular, system integrators for mobility solutions like Airbus for aviation and BMW Group for automotive rely on sophisticated supply chains for their highly complex products. One key challenge for supply chain optimization is how best to distribute parts manufacture or assembly amongst possible suppliers at different geographical locations so as to minimize the carbon dioxide emissions and, hence, the environmental impacts that arise from transporting sub-assemblies or parts between sites for further assembly into larger parts and ultimately the final product.

Today's transport options include land (road or rail), sea or air or their combination. Each of these options is associated with carbon dioxide emissions related to the nature (size and weight) of the part and the transport's start and end locations. In addition, there may be supplier constraints related, for example, to its regional or national location, time for delivery, tariffs, etc. Airbus and BMW Group are committed to reducing their scope 1 (direct) carbon dioxide emissions and ensuring a reliable and efficient supply chain for their manufacturing processes.

The objective of this problem statement is to evaluate quantum solutions for challenges in logistics for the transportation industry. This will pave the way towards more efficient and sustainable supply chains and contribute to the future of transportation products in the aerospace and automotive sectors. The following two sections briefly summarise some of the classical and possible quantum or quantum-inspired approaches.

2 Classical Optimization

2.1 Integer linear programming

This logistics optimization problem can be formulated as an integer linear programming (ILP) problem in which the unknowns are binary variables corresponding to the choice of a pair of origin and destination sites for each transportation. The objective is the total transport cost of moving parts between sites. ILP problems, in general, are known to be NP-hard and with binary variables as here constitute one of Karp's 21 NP-complete problems [1]. The canonical form of such problems for an n dimensional binary vector $\mathbf{x}$ takes the form:

$$\begin{array}{ll} \text{maximize} & \mathbf{c}^T\mathbf{x} \\ \text{subject to constraints} & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ \text{with} & \mathbf{x} \in \{0,1\}^n \end{array}$$

For the logistics problem, $\mathbf{c}$ is a vector of costs for moving parts between pairs of sites. Amongst the constraints on the binary variables is the requirement that the destination site for sub-parts matches the origin site for the resulting assembled parts and that origin and destination sites are always different.

The naive way to solve an ILP is simply to remove the constraint that x is binary or integer, solve the corresponding linear problem (called the LP relaxation of the ILP), and then round the entries of the solution to the LP relaxation. But not only may this solution not be optimal, it may not even be feasible; that is, it may violate some constraints.

2.2 Heuristic approaches

For NP-hard problems such as this, exact solutions are typically not available at sufficiently large scales because of the exponential growth of the solution space, and heuristic approaches are often adopted. One heuristic approach to this problem (not guaranteed to find the optimal solution) is using evolutionary algorithms [2]. In particular, genetic algorithms, [3], are the most popular type of evolutionary algorithm and are often applied to combinatorial optimization problems. Another heuristic approach is the use of answer set programming[4]. This is a form of declarative programming that is orientated towards complex search problems such as this. Many other approaches can be found in the literature, but no generally applicable method has been demonstrated so far for industrial-scale problems, and decisions will often be based instead on expert judgment. Hence, there is a strong interest in both Airbus and BMW in whether a quantum approach may provide a practical means of finding optimal or near-optimal solutions for logistics problems such as this.

3 Quantum Optimization

The practical application of fault-tolerant algorithms for solving optimization problems awaits developments in technology, which may be more than a decade away. In the near term, hybrid classical/quantum approaches employing variational methods, [5], are one way forward in accommodating the limitations of current Noisy Intermediate-Scale Quantum (NISQ) technology for gate-based quantum computation.

However, even these approaches have yet to demonstrate the capability to tackle optimization problems of practical importance. It is common to see quantum solution approaches applied to reduced "toy" problems with the intention that knowledge can be gained on how the solution methods will scale as the technology advances, giving an understanding of whether or when there might be a quantum advantage. In certain cases, an attempt is made to decompose industrial scale problems into smaller quantum solvable problems and tackle the larger problem as a hybrid quantum-classical composite of smaller quantum problems handled in an iterative fashion e.g., for an analogue treatment of a logistics problem see [6] and for digital approaches using circuit knitting (see e.g., [7], [8]). Very often, the quantum solver forms a small component in the overall solution process with a focus on where a quantum approach can yield the highest benefit (e.g., [9]).

3.1 Analogue quantum computing

In the analogue approach, a physical system of qubits represents the problem being modelled through the Hamiltonian for the system [10], [11]. In particular, for adiabatic quantum computing, the Hamiltonian slowly evolved from an initial form in which the ground state is known to a final form in which the ground state corresponds to the solution to a problem. Provided the evolution is adiabatic, then this provides a mechanism for recovering a solution state from a given initial ground state. The desired Hamiltonian is modelled through appropriate couplings between the logical qubits in the system. These, in turn, are mapped to the actual physical qubits through a minor embedding. The problem is usually formulated as a quadratic unconstrained binary optimization (QUBO) and then converted to an Ising Hamiltonian for a solution through minimization (see e.g., [12]). Alternatively, when working with Rydberg atom arrays, the problem has to be cast as a maximum independent set problem on unit-disk graphs [13].

3.2 Digital quantum computing

In the digital (gate-based) approach, a series of discrete gate operations are performed on the initial state of a set of qubits such that the final resulting state of the qubits encodes the solution to a given problem in a probabilistic sense. Gate operations include both single-qubit rotations and multi-qubit entangling operations such as CNOT gates. In the digital (discrete) form of adiabatic quantum computing, the action of the time-dependent system Hamiltonian is broken down into a series of short-time discrete actions through Trotterization. Each of these corresponds to alternating blocks of entangling or mixing gates in the circuit. To obtain close to optimal solutions, a large number of alternating blocks would be required, and hence, high-fidelity gate operations are necessary. For the current NISQ technology, this is not possible; thus, a heuristic variational approach is adopted. The QAOA algorithm [14], [15] utilizes just a small number of alternating blocks but introduces circuit parameters for the gates in what is known as the circuit ansatz for the problem. Through the minimization of an objective arising from the output state of the circuit, this hybrid quantum/classical approach can arrive at a near-optimal solution to the problem with a relatively shallow circuit.

3.3 Quantum inspired approaches

Quantum-inspired approaches utilize aspects of quantum physics but run on classical computing, with the goal of designing novel, efficient algorithms. The most obvious quantum-inspired approach is that of tensor networks, [16]. These are algorithms that compress and process information extremely efficiently while maintaining accuracy and reducing the required resources needed for computations. Tensor networks seem well-suited for tackling optimization problems, [17].

4 Case Study

The manufacture of complex products like cars and aircraft can be represented by a product breakdown structure (PBS) for a set P of M parts. The PBS decomposes the final product into a structured system according to different levels like the final product, parts, sub-parts, etc., as illustrated for a simple example in figure 1 or in tabular form in table 1. This shows hierarchically how component parts are assembled into larger parts. Parts at the lowest level in the PBS tree are brought together and assembled into larger parts at the next level in the tree. This is repeated for successive levels until finally, at the highest level, the end product is assembled. The PBS is represented by a set ϕ of all integer tuples (r, s) corresponding to links in the PBS where r is the index for the lower level constituent part and s is the index for the higher level resultant part. It is also useful to define the set ψ of all integer tuples (r, s) corresponding to parts r and s with $r < s$ at a certain level such that these are sub-parts of a common part at the next level up in the PBS. Thus for the example in figure 1 the sets ϕ and ψ are given by $\phi = \{(2, 1), (3, 1), (4, 1), (5, 2), (6, 2), (7, 2), (8, 3), (9, 4), (10, 4)\}$ and $\psi = \{(2, 3), (2, 4), (3, 4), (5, 6), (5, 7), (6, 7), (9, 10)\}$.

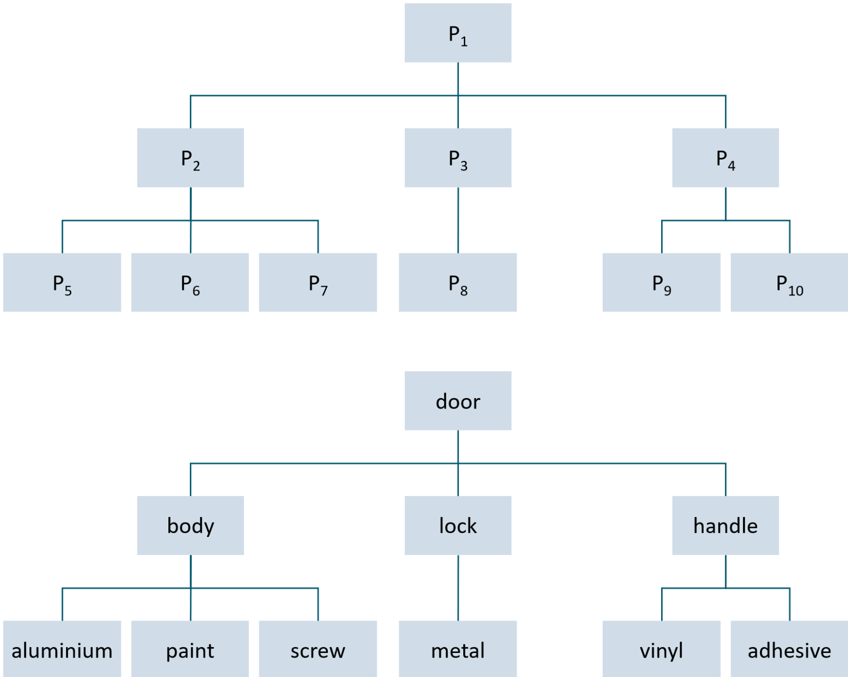

Figure 1: Example product breakdown structure for parts P_a, $a = 1, 2, ...10$ (upper) with illustration of possible parts for a door assembly (lower).

Different parts of the PBS are produced or assembled at different locations from a set S of N sites and need to be transported to different sites for further assembly into larger parts. In general, this can be described by assigning each part from the set of M parts to one site from the set of N sites through a map between the indices p of the parts and s of the sites:

$$f : p \to s, \quad f(a) \in \{1, 2, \ldots N\} \;\; \forall a \in \{1, 2, \ldots M\}.$$

It is assumed for simplicity that all sites can manufacture/assemble any of the parts. Thus, there are N^M possible ways of assigning parts to sites, though some of these will violate the constraints discussed below. A simple example of such an assignment for the PBS in figure 1 is given in table 2. Figure 2 shows the corresponding geographical locations of the sites in this simple example and the transport of

	P1	P2	P3	P4
P2	X			
P3	X			
P4	X			
P5		X		
P6		X		
P7		X		
P8			X	
P9				X
P10				X

Table 1: Tabular representation of PBS; each row is a different sub-part, each column is a different assembled part and X indicates the subpart contributes to the assembled part.

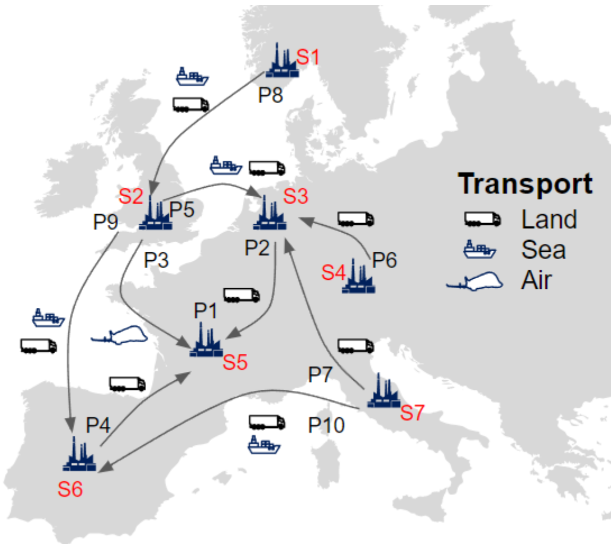

Figure 2: Example of assignment of 10 parts to 7 sites for PBS in figure 1

parts between the sites. For each part, $P_a \in P$, and for each pair of sites S_i and $S_j \in S$, there is a cost c^a_{ij} in mass of carbon-dioxide for transporting the part, P_a, between the sites S_i and S_j. There are a total of $MN(N-1)/2$ such possible travel costs assuming the direction of travel does not change the cost ($c^a_{ij} = c^a_{ji}\ \forall a, i, j$) and departure site must, of course, be different from destination site which can be imposed by setting very high transport cost if they are chosen the same ($c^a_{ii} = 10^{30}\ \forall a, i, j$). The objective, therefore is to minimise the total cost over the assignment of parts to sites, i.e., we seek the minimum total logistical cost in carbon dioxide given by:

$$C_{min} = \min_f \sum_{(r,s)\in\phi} c^r_{f(r)f(s)}$$

This problem can be formulated in terms of binary variables x_{rsij} corresponding to the assignment of origin site i and destination site j for parts r and s with $(r, s) \in \phi$. This would be the formulation for the integer linear programming technique described earlier. Alternatively, it can be formulated in terms of binary variables x_{ri} corresponding to the assignment of site i for part r in the PBS. The two

	S1	S2	S3	S4	S5	S6	S7
P1					X		
P2			X				
P3		X					
P4						X	
P5		X					
P6				X			
P7							X
P8	X						
P9		X					
P10							X

Table 2: Assignment of parts to sites for example in figure 2

alternative sets of variables are related through $x_{rsij} = x_{ri}x_{sj}$. In what follows, a formulation based on the latter choice is outlined, as this leads naturally to the QUBO formulation frequently used in quantum computing.

In terms of the site allocation variables, the objective function can be written as

$$C(x) = \sum_{(r,s)\in\phi} c_{ij}^{r} x_{ri} x_{sj}$$

This is subject to the following explicit constraints on the binary variables:

(1) One and only one assignment of the site per part:

$$\sum_{i} x_{ri} = 1 \;\; \forall r$$

(2) Origin and destination for each transport must be different:

$$\sum_{(r,s)\in\phi} \sum_{i} x_{ri} x_{si} = 0$$

(3) The origins of 2 sub-parts of a common part must be different:

$$\sum_{(r,s)\in\psi} \sum_{i} x_{ri} x_{si} = 0$$

This combination of objective and constraints can be formulated as a quadratic unconstrained binary optimization (QUBO) through the addition of penalty terms to the objective with appropriate positive values for the coefficients λ_1, λ_2 and λ_3 in

$$Q = C + \lambda_1 C_1 + \lambda_2 C_2 + \lambda_3 C_3$$

with

$$
\begin{aligned}
C_1 &= \sum_{a=1}^{M}\left(\left(\sum_i x_{ai} - 1\right)^2 - 1\right) \\
&= \sum_{a=1}^{M}\sum_i\left(-2x_{ai}x_{ai} + \sum_j x_{ai}x_{aj}\right) \\
C_2 &= \sum_{(r,s)\in\phi}\sum_i x_{ri}x_{si} \\
C_3 &= \sum_{(r,s)\in\psi}\sum_i x_{ri}x_{si}
\end{aligned}
$$

Minimisation of the objective Q will result in an optimal solution to the logistics problem. This particular formulation or any other valid formulation may be used by participants in the quantum computing challenge for this problem.

5 Submission Guidelines and Key Performance Metrics

This section provides guidance to participants for this specific problem statement. We also emphasize to follow the general submission guidelines provided on the challenge website. The followings steps are suggested to demonstrate the solution capability with respect to the logistics optimization problem.

- **Step 1: Minimum transport cost:** Determine the assignment of sites to parts that minimize the transport cost for the PBS in figure 1 given the individual transport costs between sites provided in table 3. This provides a check on the method.
- **Step 2: More sites/parts/levels:** Increase the number of sites, the number of parts (doubling in steps) and/or the number of levels (increasing linearly up to a minimum of 5) for the PBS and find the optimal solution.
- **Step 3: Additional constraints:** For any of the previous cases now require that each part should now be manufactured or assembled at two sites and determine the minimum cost.

Following these steps, it should be possible to:

- Provide an assessment of how the performance would develop as N, M, and the complexity of the PBS are varied. Provide a statement on the number of qubits required for a certain complexity level.
- Determine the maximum feasible values of N, M and PBS complexity with current hardware?

The following assessment criteria will be considered to evaluate the submissions:

- Number of problem steps successfully completed.
- Algorithmic performance (solution quality, scaling, run-time).
- Clarity and reproducibility of approach and results.

6 Appendix

The table below lists the emission costs in arbitrary units for transporting parts between pairs of sites. These costs appear in the objective function. A csv file with the data in the table along with a python script for generating the data will be provided. The script can be used to generate new data for a PBS with an increased number of sites, parts, and levels.

a	i	j	c_{aij}	a	i	j	c_{aij}	a	i	j	c_{aij}	a	i	j	c_{aij}	a	i	j	c_{aij}
2	1	2	1.64	4	1	2	8.06	6	1	2	7.31	8	1	2	2.73	10	1	2	4.9
2	1	3	1.05	4	1	3	5.14	6	1	3	4.66	8	1	3	1.74	10	1	3	3.12
2	1	4	1.09	4	1	4	5.35	6	1	4	4.85	8	1	4	1.81	10	1	4	3.25
2	1	5	1.43	4	1	5	7.03	6	1	5	6.38	8	1	5	2.39	10	1	5	4.27
2	1	6	0.91	4	1	6	4.47	6	1	6	4.06	8	1	6	1.52	10	1	6	2.72
2	1	7	1.7	4	1	7	8.32	6	1	7	7.55	8	1	7	2.82	10	1	7	5.05
2	2	3	0.59	4	2	3	2.88	6	2	3	2.61	8	2	3	0.98	10	2	3	1.75
2	2	4	0.37	4	2	4	1.8	6	2	4	1.63	8	2	4	0.61	10	2	4	1.09
2	2	5	1.12	4	2	5	5.49	6	2	5	4.98	8	2	5	1.86	10	2	5	3.34
2	2	6	0.24	4	2	6	1.18	6	2	6	1.07	8	2	6	0.4	10	2	6	0.72
2	2	7	1.79	4	2	7	8.76	6	2	7	7.95	8	2	7	2.97	10	2	7	5.32
2	3	4	0.93	4	3	4	4.58	6	3	4	4.15	8	3	4	1.55	10	3	4	2.78
2	3	5	1.02	4	3	5	4.98	6	3	5	4.52	8	3	5	1.69	10	3	5	3.02
2	3	6	1.35	4	3	6	6.62	6	3	6	6.01	8	3	6	2.25	10	3	6	4.02
2	3	7	1.65	4	3	7	8.07	6	3	7	7.32	8	3	7	2.74	10	3	7	4.9
2	4	5	1.65	4	4	5	8.12	6	4	5	7.36	8	4	5	2.75	10	4	5	4.93
2	4	6	0.19	4	4	6	0.94	6	4	6	0.85	8	4	6	0.32	10	4	6	0.57
2	4	7	1.52	4	4	7	7.45	6	4	7	6.76	8	4	7	2.53	10	4	7	4.52
2	5	6	1.04	4	5	6	5.1	6	5	6	4.63	8	5	6	1.73	10	5	6	3.1
2	5	7	0.7	4	5	7	3.41	6	5	7	3.1	8	5	7	1.16	10	5	7	2.07
2	6	7	1.57	4	6	7	7.68	6	6	7	6.97	8	6	7	2.61	10	6	7	4.67
3	1	2	5.56	5	1	2	7.66	7	1	2	1.0	9	1	2	1.49				
3	1	3	3.54	5	1	3	4.88	7	1	3	0.64	9	1	3	0.95				
3	1	4	3.68	5	1	4	5.08	7	1	4	0.66	9	1	4	0.99				
3	1	5	4.85	5	1	5	6.68	7	1	5	0.87	9	1	5	1.3				
3	1	6	3.08	5	1	6	4.25	7	1	6	0.56	9	1	6	0.83				
3	1	7	5.73	5	1	7	7.9	7	1	7	1.03	9	1	7	1.54				
3	2	3	1.98	5	2	3	2.73	7	2	3	0.36	9	2	3	0.53				
3	2	4	1.24	5	2	4	1.71	7	2	4	0.22	9	2	4	0.33				
3	2	5	3.79	5	2	5	5.22	7	2	5	0.68	9	2	5	1.02				
3	2	6	0.82	5	2	6	1.12	7	2	6	0.15	9	2	6	0.22				
3	2	7	6.04	5	2	7	8.32	7	2	7	1.09	9	2	7	1.62				
3	3	4	3.15	5	3	4	4.35	7	3	4	0.57	9	3	4	0.85				
3	3	5	3.43	5	3	5	4.73	7	3	5	0.62	9	3	5	0.92				
3	3	6	4.56	5	3	6	6.29	7	3	6	0.82	9	3	6	1.23				
3	3	7	5.56	5	3	7	7.66	7	3	7	1.0	9	3	7	1.5				
3	4	5	5.59	5	4	5	7.71	7	4	5	1.01	9	4	5	1.5				
3	4	6	0.64	5	4	6	0.89	7	4	6	0.12	9	4	6	0.17				
3	4	7	5.13	5	4	7	7.07	7	4	7	0.93	9	4	7	1.38				
3	5	6	3.51	5	5	6	4.84	7	5	6	0.63	9	5	6	0.95				
3	5	7	2.35	5	5	7	3.24	7	5	7	0.42	9	5	7	0.63				
3	6	7	5.3	5	6	7	7.3	7	6	7	0.96	9	6	7	1.42				

Table 3: Travel cost in mass of carbon-dioxide, c_{ij}^a for transporting part P_a from site S_i to site S_j

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