Quantum Solvers: Predictive Aeroacoustic & Aerodynamic modeling

Airbus - BMW Group
Quantum Computing Challenge 2024 | Forward Track
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1 Introduction

The ability to accurately predict aerodynamic flows and the propagation of sound waves within such flows are key capabilities required within the mobility sector. This ability requires the efficient solution of Partial Differential Equations (PDEs) utilising state of the art High Performance Computing to resolve complex multi-scale problems of many millions of degrees of freedom. Accurate prediction reduces the need for expensive and long lead time physical testing/prototyping which in turn enables a wider exploration of the overall design space in a given time frame. Airbus and BMW Group are seeking more efficient ways to solve PDEs so novel product solutions can be identified which meet or exceed the challenging quality and environmental requirements of their customers.

Being able to accurately predict the aerodynamic forces at the edge of the vehicle operating envelope is required for sizing of the primary structure. It can also give critical information on energy consumption in the cruise condition. Accurate prediction requires the solution of nonlinear PDEs such as the Navier-Stokes equations. Due to the limited accuracy of existing methods and limits in computational resources there is still a reliance on wind tunnel testing at these extreme conditions of high acceleration and Mach number.

Aeroacoustics holds immense importance for both sectors, aerospace and automotive. In aerospace, advancements in aeroacoustics modeling directly contribute to enhancing passenger comfort, reducing community noise pollution, and ensuring compliance with environmental regulations. In the automotive industry, progress in aeroacoustics leads to quieter cabins, improved vehicle designs, and reduced road noise, thereby enhancing the driving experience and promoting sustainable mobility solutions. Therefore, in addition to the aerodynamic efficiency, the acoustic signature is a main design criterion in automotive and aerospace industries.

The variety of numerical solutions to tackle the computational aeroacoustics (CAA) applications [1] provides researchers with a tool-set for virtual multi-physics modeling to reduce the reliance on physical testing and measurement. Coupling Computational Fluid Dynamics (CFD) models with the acoustic propagation methods enables hybrid aeroacoustics simulations to be performed. The flow is resolved using compressible or incompressible CFD, and the acoustic source terms within the flow are propagated through the mean flow using acoustic wave propagation.

This problem statement focuses on finding a quantum based approach to solve the relevant aerodynamic and acoustic equations. If a potential future quantum advantage is discovered this could transform our future aerodynamic and acoustic modeling capabilities.

2 Governing Equations

This section describes the PDEs that are relevant to the calculation of steady inviscid compressible flow and the propagation of sound in such a flow. For the purposes of the challenge the most complex equation set to consider is the Euler Equation. From this, the simplest equation to consider is derived which is the wave equation.

Fluid motion is governed by the Navier–Stokes equations [2], a set of coupled and nonlinear partial differential equations derived from the basic laws of conservation of mass, momentum and energy. Taking the assumptions of inviscid flow, the Navier-Stokes equations are reduced to the Euler equations which, in their 2D form, can be expressed as:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} &= 0, \\ \rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x}, \\ \rho\frac{\partial v}{\partial t} + \rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y}, \\ \rho\frac{\partial h_t}{\partial t} + \rho u\frac{\partial h_t}{\partial x} + \rho v\frac{\partial h_t}{\partial y} &= 0, \end{aligned} \tag{1}$$

where x and y are coordinates in the Cartesian reference frame, t is time, ρ is the density, u and v are the velocity components in the x and y directions, p is the static pressure and h_t is the enthalpy. Closure is achieved through application of the equation of state and the definition of stagnation enthalpy and Mach number:

$$\begin{aligned} P &= \rho RT, \\ h_t &= c_p T + \frac{1}{2}|V|^2, \\ M &= \frac{|V|}{c}, \end{aligned} \tag{2}$$

where P is the total pressure, R is the gas constant, T is the temperature, c_p is the heat capacity at constant pressure. M is the Mach number, V is the freestream velocity, γ is the ratio of heat capacities at constant pressure and constant volume, and c is the speed of sound given by $c = \sqrt{\gamma RT}$.

To model acoustic disturbances within a given steady flow solution a separation is made into a base flow (bar) component and a fluctuation (prime) component as follows:

$$\begin{aligned} \rho &= \bar{\rho} + \rho', \\ u &= \bar{u} + u', \\ v &= \bar{v} + v', \\ p &= \bar{p} + p', \\ h_t &= \bar{h_t} + h_t'. \end{aligned} \tag{3}$$

Inserting the relation above into the Euler equations and dropping the prime for convenience, leads to the linearised Euler equations:

$$\begin{aligned} \frac{\partial p}{\partial t} &= -\bar{\rho}c^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \bar{u}\frac{\partial p}{\partial x} - \bar{v}\frac{\partial p}{\partial y} - p\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y}\right), \\ \frac{\partial u}{\partial t} &= -\frac{1}{\bar{\rho}}\frac{\partial p}{\partial x} - \bar{u}\frac{\partial u}{\partial x} - u\frac{\partial \bar{u}}{\partial x} - \bar{v}\frac{\partial u}{\partial y} - v\frac{\partial \bar{u}}{\partial y} - \frac{p}{\bar{\rho}c^2}\left(\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}\right), \\ \frac{\partial v}{\partial t} &= -\frac{1}{\bar{\rho}}\frac{\partial p}{\partial y} - \bar{v}\frac{\partial v}{\partial y} - v\frac{\partial \bar{v}}{\partial y} - \bar{u}\frac{\partial v}{\partial x} - u\frac{\partial \bar{v}}{\partial x} - \frac{p}{\bar{\rho}c^2}\left(\bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial x}\right). \end{aligned} \tag{4}$$

The linearised Euler equations describe the propagation of acoustic waves in an arbitrary base flow (which can be calculated using the Euler equations). If the base flow is homogeneous the derivatives of $\bar{u}$ and $\bar{v}$ are zero and drop out of the equation. If we consider the flow to be in the x direction only then $\bar{v} = 0$ and the equations reduce to the form:

$$\begin{aligned} \frac{\partial p}{\partial t} &= -\bar{\rho}c^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \bar{u}\frac{\partial p}{\partial x}, \\ \frac{\partial u}{\partial t} &= -\frac{1}{\bar{\rho}}\frac{\partial p}{\partial x} - \bar{u}\frac{\partial u}{\partial x}, \\ \frac{\partial v}{\partial t} &= -\frac{1}{\bar{\rho}}\frac{\partial p}{\partial y} - \bar{u}\frac{\partial v}{\partial x}. \end{aligned} \tag{5}$$

If the sound source is in quiescent air then all the $\bar{u}$ and $\bar{v}$ terms are zero which gives the acoustic wave equation:

$$\begin{aligned} \frac{\partial p}{\partial t} &= -\bar{\rho}c^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right), \\ \frac{\partial u}{\partial t} &= -\frac{1}{\bar{\rho}}\frac{\partial p}{\partial x}, \\ \frac{\partial v}{\partial t} &= -\frac{1}{\bar{\rho}}\frac{\partial p}{\partial y}. \end{aligned} \tag{6}$$

By changing from the cartesian (x, y) to the polar coordinate frame of reference where r is the distance from the point source this simplifies further to:

$$\frac{\partial^2 rp}{\partial t^2} - c^2\frac{\partial^2 rp}{\partial r^2} = 0 \tag{7}$$

where:

$$r^2 = x^2 + y^2$$

Equation 7 is the simplest of the PDEs that we will consider for the challenge.

3 Classical Numerical Approaches

Predicting the aerodynamic and aeroacoustic properties of an airflow requires a combination of flexible and robust numerics. There are two complementary approaches to solve respective sets of the relevant PDEs as outlined above: Numerical discretization techniques and deep learning solutions.

Discretization Techniques

Predicting the properties of fluid flows and noise propagation can be addressed with Finite Volume Methods (FVM), especially when dealing with complex geometries; and at the same time low-dispersion and low-dissipation numerics - typical of high-order Finite Element Methods (FEM).

Both FEM and FVM represent the grid-based techniques, which interpolate the initial continuous coupled system of Partial Differential Equations (PDE) on a grid of points. The main difference between them lies in the way they represent the exact solution by an approximate one and how this approximate solution satisfies the PDE. Even though these techniques are considered as conventional for industrial applications, they still lack efficiency while tackling complex multi-physics simulations. Some generic properties of the methods mentioned above regarding different numerical criteria are shown in Table 1 [3].

Numerical method	Complex geometries	High-order accuracy and *hp*-adaptivity	Explicit semi-discrete form	Conservation laws	Elliptic problems
Finite Differences	-	+	+	+	+
Finite Volumes	+	-	+	+	(+)
Finite Elements	+	+	-	(+)	+
Discontinuous Finite Elements	+	+	+	+	(+)

Table 1: Generic properties of the most widely-used numerical methods ('+' represents success, '-' - short-coming in the method, '(+)' - method can be used but it remains the less natural choice).

Using discrete numerical solutions allows mapping the initial continuous problem to a linear system of equations with a large sparse matrix to be inverted. The matrix inversion step makes those methods extremely heavy for application to realistic models at a high accuracy level. To solve nonlinear Euler equations a time stepping scheme can be used together with a linear solver [4].

Deep Learning Techniques

Besides this discrete technique, deep learning solutions and tools have become popular in the context of physical simulations recently. [1].

They can be based on e.g. on surrogate modeling, by building an approximate representation of the solution by means of an artificial Neural Network (NN). From this perspective, solving the PDEs is turned into a regression problem for which classical machine learning algorithms can be applied. Besides the discrete techniques, Deep Learning solutions and tools have become popular in the context of physical simulations.

Deep Learning methods have the potential to fundamentally change the vision of classical numerical

[1] Physics-based Deep Learning https://physicsbaseddeeplearning.org/intro.html

modelling by improving the accuracy of conventional discrete simulations. However, they still involve multiple fundamental challenges to overcome, namely the high numerical cost in the scale of industrial data. Note that both discrete and deep learning techniques require high computational power in many relevant scenarios. This makes methods such as CFD and CAA very time and resource consuming even for sophisticated HPC clusters.

Note that both discrete and deep learning techniques require high computational power in many relevant scenarios. This makes disciplines such as Computational Fluid Dynamics (CFD) very time and resource consuming tasks, and challenging even for sophisticated High Performance Computing (HPC) clusters.

4 Quantum Approaches for Solvers

Here we give a brief and non-exhaustive overview of quantum approaches to solve PDEs. Since unitary quantum operations on a Hilbert space are linear by nature the solution of a non-linear problem with quantum computing requires some form of map to a linear problem. A number of approaches have been adopted in the literature for linearising partial differential equations making them amenable to quantum solution through linear solvers. These include, amongst others, a mean field approach [5], a Carleman linearisation [6] and a Liouville embedding [7]. With the adoption of a time stepping procedure such as Euler forward differencing, the problem is reduced to a linear system of algebraic equations.

Quantum Linear Solvers

One of the earliest and well-known quantum algorithms for solving linear systems of equations was that of Harrow, Hassidim, and Lloyd [8]. This achieves an exponential improvement in complexity (i.e. run-time) with respect to problem size compared to classical algorithms. Subsequently an improved complexity with respect to precision was obtained in the quantum algorithm of Childs et al. [9]. This was achieved by replacing the quantum phase estimation of [8] with an approach based on the quantum singular value transformation (QSVT). The algorithm of Childs et al. can be seen as a special case of the more general QSVT algorithm of Gilyen et al. [10]. It should be noted that any potential exponential improvement is at risk due to state preparation or state readout requirements [11]. This needs to be addressed in some form without the use of "passive QRAM" for which there is no known scalable physical implementation [12].

However, given the current Noisy Intermediate-Scale Quantum (NISQ) devices' limitations, the proposed algorithms (HHL and QSVT) are still a long-term goal, characterized by large circuit depths, i.e. very big numbers of gate operations and hence a requirement for very high fidelities. This can be achieved partly through higher fidelity in the hardware but also mainly through the use of error correction requiring many physical qubits for each logical qubit. This possibility is still many years away with both the gate fidelity and the number of physical qubits falling far short of requirements.

Variational Quantum Methods

In the near-term with the current Noisy Intermediate-Scale Quantum (NISQ) devices, there have been some advancements in the development of variational algorithms for solving the linear systems of equations - Variational Quantum Linear Solver (VQLS) [13], and for finding the spectrum of eigenvalues and eigenvectors for a density matrix - Variational Quantum State Diagonalisation (VQSD) [14]. Both algorithms demonstrate good robustness to the NISQ restrictions, although they lack an efficient state preparation routine to decompose the initial classical matrices into quantum states' products. It is an essential step if there is to be any possibility that the algorithm may have quantum advantage with respect to its classical counterparts. These variational approaches to linear solvers could be included in the workflow starting with linearization as described above.

Tensor Networks

Other variational approaches avoiding the direct use of linear solvers include the use of tensor networks applied to differential equations (see as examples [15] and [16];), employing multiple copies of variational quantum states [17], and the physics informed neural network (PINN) inspired approach

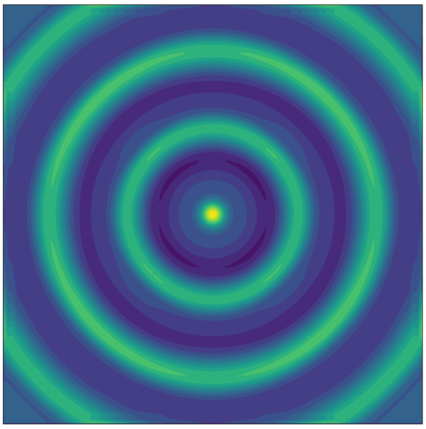

Figure 1: Contour plot of the pressure field around a stationary point source.

of Differentiable Quantum Circuits (DQC). These result from an analytical derivation of the functions defined as expectation values of the parameterised quantum circuit, thus avoiding discretization error accumulation [18]. The DQCs are trained to satisfy PDEs and specific boundary conditions.

5 Case Study

We wish to understand if and how Quantum Computing can offer a significant advantage in the solution of linear and nonlinear PDEs tailored to the specific needs of mobility industry challenges. Recent progress shows promising advances which we would like to explore for two problem cases: noise transmission through an airflow, and airflow prediction over an aerofoil. The proposed solution should be applicable to at least one problem case. It is advantageous if the solution can tackle both problem cases or is at least in principle scalable to the other case.

Problem Case 1: Noise Transmission through Air.
Noise generated by an aircraft or automobile physically corresponds to a longitudinal pressure wave which propagates through air. The noise source to be studied in the problem shall be a single point source placed in a 2D spatial domain. The acoustic source will be a sinusoidal wave-form and amplitude $A = 1dB$. The following set of acoustic frequencies shall be considered: $f_1 = 100\text{Hz}$, $f_2 = 1000\text{Hz}$, and $f_3 = 10000\text{Hz}$. The 2D source is in quiescence - i.e. base flow $\bar{u} = \bar{v} = 0$. For the purpose of demonstration the ambient conditions of interest are: $T_0 = 288.15\text{K}$, $P_0 = 113920\text{Pa}$, $\rho = 1.2250\text{kg/m}^3$. For these conditions the speed of sound can be calculated as $c = 340.2\text{m/s}$. The problem shall be resolved without any reflection effect from the boundaries (e.g. equivalent to numerical non-reflective boundary conditions). An illustration of the predicted pressure disturbance around a point source in quiescent air is given in Fig. 1.

The relevant PDEs in the cartesian or polar reference frames are given in Eq. 6 or Eq. 6, respec tively. This is the problem that participants may wish to first focus on using quantum computing methods.

The quantum application should then be extended to include the effect of a homogeneous base flow. This represents a point source in an air flow and therefore the propagation of the noise generated by a moving vehicle to an external observer. This is a key capability to determine noise impact on the community near an airport or road. An illustration of the predicted pressure disturbance around a point source in moving air is given in Fig. 2.

This breaks down the symmetry of the problem and so the relevant PDEs are Eq. 5. For the

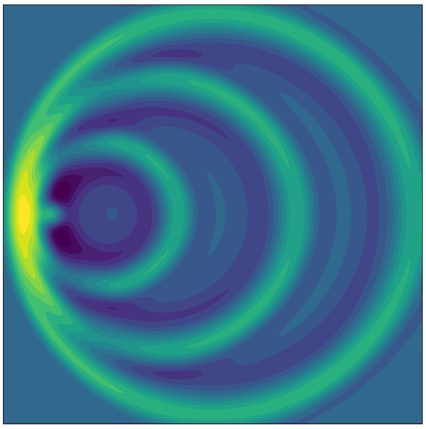

Figure 2: Contour plot of the pressure field around
a point source in a homogeneous base flow.

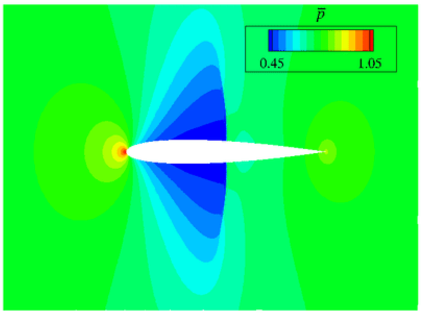

Figure 3: Contour plot of pressure around a NACA0012 aerofoil

purpose of the demonstration the baseflow conditions are $\bar{u} = 272.16\text{m/s}$ and $\bar{v} = 0$. The same ambient condition expressed previously can be used.

Problem Case 2: 2D Aerofoil Flow Prediction.

A modern wing generates lift by accelerating the air on the upper surface to a localised supersonic flow creating a suction pressure force. This supersonic flow is terminated by a shock wave which contributes to the wing drag force. The wing profile / aerofoil shaping plays a key role in minimising the drag force at the desired lift.

The creation of the supersonic flow and the presence of the shock waves are due to the compressible nature of air at transonic speeds and the prediction of these features is critical to ensure an efficient wing design. Due to the non-linear nature of the flow in this regime the complete aerofoil flow must be computed. If we assume a compressible but inviscid flow (for simplicity) the relevant equations in 2D are given in Eq. 1. These are the most complex equations to solve using quantum methods due to their nonlinear characteristics.

For the purpose of the demonstration we will consider the NACA0012 aerofoil shape as this exhibits the relevant flow features described above at transonic Mach numbers. An example euler CFD flow field from [19] is illustrated in Fig. 3.

The external shape of the NACA0012 aerofoil in the cartesian frame of reference (x, y) is given in [20] and may be calculated using the following equation.

$$y = \pm A(Bx^{0.5} - Cx - Dx^2 + Ex^3 - Fx^4) \tag{8}$$

where:

$$A = 0.594689181, B = 0.298222773, C = 0.127125232,$$
$$D = 0.357907906, E = 0.291984971, F = 0.105174606.$$

Since there can be no flow through the aerofoil surface the velocity normal to any point on the aerofoil surface is zero. However since we use the inviscid assumption the tangential flow at any point on the surface can be non-zero. For the purpose of demonstration the conditions of interest are: $T_0 = 288.15$K, $P_0 = 113920$Pa, $\rho = 1.2250$kg/m^2, and $c = 340.2$m/s. In the far field, well away from the body, all flow variables should be uniform and correspond to the freestream conditions. The problem shall be resolved without any reflection effect from the boundaries (e.g. equivalent to numerical non-reflective boundary conditions). To ease the computational challenge the aerofoil incidence is 0.0 degrees therefore the flow can be considered symmetric about $y = 0$.

6 Submission Guidelines and Key Performance Metrics

This section provides guidance to participants for this specific problem statement. We also emphasize to follow the general submission guidelines provided on the challenge website. We recommend to consider the following criteria which will be employed for the evaluation:

- The designed algorithm is expected to appropriately utilise a significant portion of quantum components, either quantum inspired elements or quantum circuit / analogue solutions.
- The motivation and potential advantage of the quantum solution shall be demonstrated in fair comparison with classical alternatives.
- The algorithm should be implemented in a common programming language (e.g. python) and applicable to at least one problem case. It is advantageous if the solution can tackle both problem cases or is at least in principle scalable.
- The feasibility of the algorithm shall be demonstrated. This can be either implemented by quantum simulation, emulation or use of quantum hardware.
- The quality of the solution shall be evaluated. Standard numerical tools can be used for the comparison.
- The computational performance of the methods and also studies on the (speed) computational complexity of the algorithm are important. An analysis of the requirements on the quantum resources needed as well as the scalability to large problem sets shall be discussed. This shall include number of mesh points and number of variables.
- Participants should provide an assessment on how well the algorithm generalizes to other problems.

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